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1	Development of a Negative Stiffness Friction Damping Device with an
2	Amplification Mechanism
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7	ABSTRACT: This paper proposes an innovative negative stiffness friction damping device
8	(NSFDD), which is characterized with a combination of negative stiffness and friction damping
9	force. The large force capacity of the NSFDD is realized by the amplification mechanism of a two-
10	consecutive lever and using the pre-compressed gas springs. A mechanical model is developed to
11	represent the force-displacement relationship of the proposed NSFDD. The influence of key
12	parameters on the NSFDD performance is analyzed using this mechanical model and demonstrates
13	the advantage of using pre-compressed gas springs. A NSFDD specimen was manufactured and
14	examined by experimental tests. The NSFDD specimen exhibits an obvious negative stiffness and
15	stable energy dissipation, demonstrating the feasibility and effectiveness of the NSFDD. Finally,
16	a refined finite element (FE) model is developed to represent the NSFDD. Both the experimental
17	tests and FE simulation validate the accuracy of the mechanical model of the NSFDD.
18	Keywords: Negative stiffness; Friction damping; Amplification mechanism; Mechanical model;
19	Experimental test.

20

# 21 **1 Introduction**

The objective of seismic protection is to reduce the force demand and control the lateral deformation of a structure subjected to strong ground motions. In the last few decades, the vibration control techniques have been developed for control the seismic responses of buildings and infrastructures [1,2]. Extensive achievements have been made in this field, such as base isolation and energy dissipation technologies. Among these technologies, negative stiffness
vibration control shows a unique characteristic, which decreases the entire stiffness of the system
to reduce acceleration response [3]. Negative stiffness means that a force is introduced to assist
motion, not to oppose it [4]. The concept of negative stiffness was first proposed by Molyneaux
[5] in 1957. Based on the mechanisms of negative stiffness, existing negative stiffness technologies
are categorized into the following types: negative stiffness metamaterials [6], buckled beams [7],
pre-compression springs [8], magnets [9], and magnetorheological technology [10], etc.

33 Thus far, the negative stiffness technology has been extensively developed in mechanical engineering [11,12] and materials science fields [13-16]. Wang et al. [17,18] designed a variety of 34 novel multistable metamaterials which were based on mechanisms in the biological organs. The 35 materials have negative stiffness effects, snap-through behaviors, higher mechanical performance 36 and lower density. The sleeve-type negative stiffness [19,20] is a novel type negative stiffness 37 metamaterial that can dissipate mechanical energy even if the metamaterial is only composed of 38 one negative stiffness cell, having a wide range of application prospects in many engineering fields. 39 Meng [21] presented negative stiffness oscillators that were realized through Euler buckled beams 40 41 and showed great potential to construct tunable broadband vibration absorption configurations. Robertson [22] used the arrangement of magnets to produce a negative stiffness element to reduce 42 the natural frequency of the system. Mehreganian [23] investigated the negative stiffness 43 honeycomb metamaterials made of double curved beams and the desired negativity in the stiffness 44 matrix could be achieved with high bistability ratios. Anastasio et al. [24-27] tested a negative 45 stiffness oscillator to exploit its linear and nonlinear dynamics and chaotic motion as well. The 46 oscillator exhibits a strong nonlinear behavior, mainly due to its polynomial elastic restoring force 47 48 with a negative stiffness region. In addition, the results show that negative stiffness has the advantages, such as damping augmentation and reduced acceleration transmissibility. 49

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In the civil engineering field, Iemura et al. [10,28,29] introduced the negative stiffness

technique to control structural dynamic responses by reducing structural stiffness and adding 51 damping. They developed a magnetorheological fluid-based negative stiffness damper and 52 53 negative stiffness friction damper (which works in a similar way to friction pendulum sliding). Das [30] proposed the shape memory alloy (SMA)-based nonlinear energy sink with negative 54 stiffness and friction. A number of parametric studies demonstrate its enhanced and robust 55 performance. The negative stiffness technology can improve the vibration isolation performances 56 [31-33] and the simulation results indicate that vibration isolators with negative stiffness could 57 have smaller displacement amplitude and force transmissibility peaks, and a wider isolation 58 frequency band. Cimellaro [34] proposed a three-dimensional (3-D) base isolation system with a 59 negative stiffness device, which could reduce the vertical acceleration and the input energy 60 transferred to the superstructure. Javanbakht [35-37] proposed an analysis model for the negative 61 stiffness dampers which was employed to analyze the response control of stay cables, 62 demonstrating the effectiveness of the negative stiffness technique for cable control. Walsh et al. 63 [38,39] proposed a variable negative stiffness device used to improve seismic protection by 64 apparent weakening, and investigated the effect of supplemental damping in the structure on the 65 66 control performance of the systems. Pasala and Sarlis [3,4,40,41] proposed a negative stiffness device with an amplification mechanism that effectively amplifies the negative stiffness. 67 Furthermore, a viscous damper was added along with the negative stiffness device to control the 68 69 displacement response of the structure induced by stiffness reduction. Nagarajaiah et al. [42-47] investigated the dynamic responses of various types of structural system (multi-story steel frame, 70 71 bridge, tall building with damped outrigger) incorporated with the negative stiffness devices. Both experimental tests and numerical simulation indicate that the control of displacement and 72 73 acceleration responses, as well as the possibility of structural damage, can be achieved by a combination of negative stiffness and supplemental damping. Antoniadis [48-50] proposed 74 KDamping which used negative stiffness to indirectly increase the inertia effect of the additional 75

mass. Peng [51] proposed a negative stiffness damping mechanism that could provide negative
 stiffness and friction damping simultaneously.

78 While the various types of negative stiffness devices have been developed, the force capacity 79 of the devices is relatively small compared to the large force demand in civil engineering. To this end, this paper proposes a novel passive negative stiffness friction damping device (NSFDD) 80 which includes a two-consecutive lever mechanism to amplify the negative stiffness and friction 81 damping force significantly. The major contribution of this paper is to develop a mechanical model 82 83 of this device and validate its cyclic performance. Section 2 presents the mechanism of the devices, including the generation of negative stiffness and friction damping force, and the amplification 84 mechanism of the two-consecutive lever. Section 3 develops the mechanical model of the NSFDD. 85 Using this model, the influence of key design parameters of the devices are assessed using 86 parametric analysis. Section 4 describes experimental tests of the NSFDD subjected to cyclic 87 lateral loads to examine its hysteretic behavior, negative stiffness, and energy dissipation capacity. 88 Finally, a finite element model is developed for this device. Both the finite element analyses and 89 experimental results validate the proposed mechanical model of this device. 90

## 91 **2** Mechanism of the NSFDD

# 92 **2.1** Principle of negative stiffness and friction damping

Existing passive negative stiffness devices are primarily compressed springs driven by a 93 specific device to generate negative stiffness <sup>[4]</sup>. However, the disadvantage of these devices is that 94 additional damping devices need to be connected to provide damping and control the displacement 95 response that would be enlarged due to reduced stiffness. To produce both negative stiffness and 96 damping simultaneously, the key component of the NSFDD is designed, as illustrated in Fig. 1. As 97 98 shown in Fig. 1(a), the ends A and B of the rotating rod are hinged to the friction plate and the upper part, respectively. The upper part only moves vertically and is always compressed by the 99 vertical force P. The force P compresses the friction plate through the rotating rod. The friction 100

plate is in contact with the friction surface and can move horizontally. When the friction plate generates displacement *x*, as depicted in Fig. 1(b), the rotating rod is driven to rotate. The vertical force *P* generates the force  $P_{\alpha}$  along the rotating rod, as presented in Fig. 1(c).  $P_{\alpha}$  generates vertical force  $P_{v}$  (i.e.  $P_{v}$  generates friction force with the friction surface) and horizontal force  $P_{h}$  (i.e. negative stiffness force). The lateral restoring force *F* of the friction plate of the key component is formulated as Eq. (1) and the force-displacement relationship is presented in Fig. 1(d).

$$F = \operatorname{sgn}(\dot{x})\mu P_{\rm v} - P_{\rm h} = \operatorname{sgn}(\dot{x})\mu P_{\alpha}\cos\alpha - P_{\alpha}\sin\alpha \tag{1}$$

107 where  $P_{\alpha} = P/\cos\alpha$ ;  $\alpha$  is the angle between the rotating rod and the initial state line,  $\alpha = \arcsin\frac{x}{S}$ , S

is the length of the rotating rod;  $\mu$  is the friction coefficient between the friction plate and the friction surface; and  $\dot{x}$  is the velocity of the NSFDD.



(c) Force diagram of working state (d) Force-dia





Fig. 1 Schematic drawing of key components of the NSFDD

## 111 **2.2 Amplification mechanism**

According to Eq. (1), negative stiffness and friction damping are proportional to the vertical compressive force *P*. To meet the response control demand of a building structure, the magnitude

of the force *P* should be large enough to yield sufficient negative stiffness and friction damping. 114 Using a mechanism to amplify the force P is an effective solution to improve the mechanical 115 116 performance of the NSFDD. In this study, the two-consecutive lever is adopted as the amplification mechanism. 117

118 The two-consecutive lever is a mechanical device in which two levers are superimposed to achieve twofold amplification of vertical force, as depicted in Fig. 2(a). Due to the overlapping 119 lever arms of the first-stage lever and the second-stage lever, the horizontal space is considerably 120 121 reduced (compared with the classical lever). For an example as illustrated in Fig. 2, amplifying the vertical force by 15 times, the horizontal space of the two-consecutive lever is approximately 7.4-122 unit lengths, while the classical lever is approximately 13-unit lengths. The reduction of the 123 occupied horizontal space is essential for application of this device in building structures. 124



(a) Schematic of the two-consecutive lever

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#### 2.3 The mechanism of the NSFDD with an amplification mechanism 126

Combining the key component in Section 2.1 with the two-consecutive lever in Section 2.2, 127 a NSFDD with an amplification mechanism is designed as shown in Fig. 3(a). The NSFDD is 128 composed of a top plate, base plate, restraint frame, friction plate, rotating rod (I and II), and two-129 consecutive lever (first-stage lever and second-stage lever). Points C and K are the fulcrum of the 130 two levers; points E and B are the resistance points of the two levers; points D and M are the power 131 points of the two levers. The lever rotates freely with the fulcrum (points C and K). At the initial 132

Fig. 2 Comparison of the lever mechanisms

133 state, the force P acts vertically on point M. The vertical force amplified by the first-stage lever is 134  $P_1$ . The force  $P_1$  acts vertically on point D through the rotating rod I (because the rotation angle of 135 rotating rod I is extremely small during the working state, it is assumed that rotating rod I vertically 136 compresses point D). The vertical force that is further amplified by the second-stage lever is 137 denoted as  $P_2$ . The force  $P_2$  acts vertically on point A through the rotating rod II to compress the 138 friction plate.

139 The mechanical model for the restoring force  $F_n$  of the friction plate of the NSFDD with the 140 two-consecutive lever is given by:

$$F_{\rm n} = n \left( {\rm sgn}(\dot{x}) \mu P_{\rm v} - P_{\rm h} \right) \tag{2}$$

where  $n=L_1L_3/(L_2L_4)$  is the amplification coefficient of the two-consecutive lever,  $L_1$ ,  $L_2$  are the lengths of the long and short arms of the first-stage lever, respectively; and  $L_3$ ,  $L_4$  are the lengths of the long and short arms of the second-stage lever, respectively.

144 Under one cycle of loading, the hysteresis curve is generated by the friction plate of NSFDD,145 the process for which is as follows:

(1) As depicted in Fig. 3(b), when performing positive loading, the friction plate moves from the initial state to the positive displacement and drives the rotating rod II to rotate to the right. The force along the rotating rod II generates horizontal negative stiffness force and vertical force associated with friction force at the interface. By summarizing the horizontal negative stiffness force and friction force, the force-displacement relationship in the positive loading direction is shown in Fig. 3(c).

(2) As depicted in Fig. 3(d), when performing positive unloading, the friction plate moves
from the maximum positive displacement to the initial state and drives the rotating rod II to rotate
to the left, generating the force-displacement relationship shown by the blue arrow in Fig. 3(e).

(3) As depicted in Fig. 3(f), when performing negative loading, the friction plate moves from
the initial state to the negative displacement and drives the rotating rod II to rotate to the left,

157 generating the force-displacement relationship shown by the red arrow in Fig. 3(g).

(4) As depicted in Fig. 3(h), when performing negative unloading, the friction plate moves
from the maximum negative displacement to the initial state and drives the rotating rod II to rotate
to the right, generating the force-displacement relationship shown by the blue arrow in Fig. 3(i).



(a) Schematics of the NSFDD



(b) Positive loading state



(c) Force-displacement relationship

for positive loading



(d) Positive unloading state



(f) Negative loading state



(h) Negative unloading state



(e) Force-displacement relationship

for positive unloading



(g) Force-displacement relationship

for negative loading



(i) Force-displacement relationship for

negative unloading

161 Fig. 3 Schematics and working principle of the NSFDD with an amplification mechanism

As shown in Fig. 3(a), point B of the rotating rod II moves downward when the NSFDD has horizontal displacement (i.e., gray dashed line). The vertical geometric displacement of point B is calculated as  $x_1$ ,  $x_1 = S - \sqrt{S^2 - x^2}$ . Because the lever principle amplifies the force and the vertical displacement, the vertical displacement  $x_3$  at the point M is:

$$x_3 = nx_1 \tag{3}$$

As the vertical force P is commonly provided by the compressed spring, the vertical 166 displacement x<sub>3</sub> at the loading point M may cause a rapid decrease in the compressed spring force. 167 Fig. 4 depicts the installation of a NSFDD in a floor of a building structure. The top plate and 168 friction plate of the NSFDD are fixed to the upper and lower floor, respectively. In a positive 169 displacement, which is defined as the lower floor moving to the right relative to the upper floor, 170 171 the friction plate is forced to move to the right, causing the rotating rod II to rotate to the right (as 172 illustrated in Fig. 4 (b)). In this condition, the pre-compressed spring moves downward, which results in a release of the pre-compressed deformation of the spring (i.e. x<sub>3</sub>, as indicated in the red 173 line in Fig. 4). Similarly, the deformation of the elements of NSFDD is also demonstrated in Fig. 174 175 4(d) at a negative displacement (i.e., the lower floor moves to the left relative to the upper floor). Therefore, the selection of suitable springs that remain stable compressive force within a range of 176 deformation is vital for ensuring robust mechanical properties of the NSFDD. 177







Fig. 4 Deformation mechanism of the NSFDD

# 179 **3. Mechanical model and parametric analysis of NSFDD**

# 180 **3.1 Compressed springs**

In this section, the mechanical properties of a mold coil spring and gas spring are compared in light of the effectiveness of use in the NSFDD. A mold coil spring is made of chromium alloy steel with a rectangular section (see Fig. 5 (a)). Most existing negative stiffness devices use the mold coil springs, as they offer greater stiffness and durability than normal springs.

185 The compressed force of a mold coil spring,  $N_{\rm m}$ , is calculated as:

$$N_{\rm m} = k_{\rm m} \vartriangle x \tag{4}$$

186 where  $k_{\rm m}$  is the mold coil spring stiffness and  $\Delta x$  is the pre-compressed deformation.

The gas spring is a new type of spring element in which high-pressure gas is sealed within a cylinder by a piston rod. When the piston rod is forced to compress the high-pressure gas, the gas spring gains elastic stiffness from the reaction force-deformation of the high-pressure gas. A unique characteristic of a gas spring is a large initial compressive force, which is transferred from the contact force of the cylinder once the piston has an extremely small displacement (see Fig. 5 (b)). The pre-compressed gas spring force <sup>[51]</sup>,  $N_g$ , can be calculated as:

$$N_{\rm g} = k_{\rm g\Delta} x + b \tag{5}$$

where  $k_g$  is the gas spring stiffness; and b denotes the initial compressive force. 193



(a) Mold coil spring

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Fig. 5 Diagram of two types of springs

195 Fig. 6 depicts the force-displacement relation of a mold coil spring and a gas spring that have identical stiffness. As the initial compressive force occupies a large portion of the total force of the 196 197 gas spring, the force variation results from the displacement change occupies a smaller percentage 198 of the total compressed force, compared to the mold coil spring. This property makes the gas spring 199 more suited for use in NSFDD, because it can remain large compressive force within a variation 200 of deformations. A quantitative analysis of this property will be presented in the following subsection. Except for this, the gas springs have another two advantages. First, with increased 201 pressure of a gas, the force capacity of a gas spring can reach several times that of the mold coil 202 203 spring if both springs have similar geometric dimensions. Second, a gas spring can remain stable mechanical properties even after a million cycles of loading, which enable it a long service life 204 against fatigue. 205



Fig. 6 Comparison of spring force between gas springs and mold coil springs

### 207 3.2 Mechanical model

## 208 **3.2.1 Mold coil spring**

If using the mold coil spring provides the pre-compressed force *P*, the mechanical model of the NSFDD can be obtained by substituting Eq. (4) into Eq. (2), given by:

$$F_{\rm m} = n\,{\rm sgn}(\dot{x})\mu P_{\rm vm} - nP_{\rm hm} = n\left(k_{\rm m}\left[\Delta x - n\left(S - \sqrt{S^2 - x^2}\right)\right]\right) \times \left({\rm sgn}(\dot{x})\mu - \frac{x}{\sqrt{S^2 - x^2}}\right) \tag{6}$$

211 Fig. 7 plots a hysteretic response of an example NSFDD which is obtained by Eq. (6) and using assumed typical values for the parameters (i.e., n = 15,  $\mu = 0.3$ ,  $k_g = 92$  kN/m, S = 60 mm, 212 and  $\Delta x = 100$  mm). As indicated in Fig. 7(a), the horizontal force P<sub>hm</sub> generated by the mold coil 213 springs provides negative stiffness at small displacements in the Oh1-Ah1 and Oh1-Bh1 phases. At 214 large displacements, because the rotation of the levers reduces the pre-compressed deformation of 215 216 the mold coil spring and the corresponding compressed force P, negative stiffness at the larger displacements in A<sub>h1</sub> and B<sub>h1</sub> is not significant. As depicted in Fig. 7(b), the vertical force  $P_{\rm vm}$ , 217 218 generated by the mold coil spring, compresses the friction plate, causing friction damping when the friction plate moves. Similarly, at large displacements (e.g., points Av1, Bv1, Cv1, and Dv1 in Fig. 219 7(b)), the pre-compressed deformation and corresponding force of the mold coil spring are 220 significantly reduced, leading to an obviously reduced friction damping force. With a combination 221 of the negative stiffness part and friction damping part, the total hysteretic response of the NSFDD 222 is shown in Fig. 7(c). At larger displacements (e.g., points A<sub>1</sub>, B<sub>1</sub>, C<sub>1</sub>, and D<sub>1</sub>), the performance 223 becomes unsatisfactory, because the pre-compressed spring force of the mold coil spring is 224 225 significantly decreased.



 $(generated by P_{hm}) \qquad (generated by P_{vm}) \qquad NSFDD$ 

Fig. 7 Force-displacement relationship of the NSFDD using mold coil springs

## 227 **3.2.2 Gas spring**

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If using the gas spring to provide the pre-compressed force *P*, the mechanical model of the NSFDD can be obtained by substituting Eq. (5) into Eq. (2), given by:

$$F_{\rm g} = n\,\mathrm{sgn}(\dot{x})\mu P_{\rm v} - nP_{\rm h} = n \left[ k_{\rm g} \left[ \Delta x - n \left( S - \sqrt{S^2 - x^2} \right) \right] + b \right] \times \left( \mathrm{sgn}(\dot{x})\mu - \frac{x}{\sqrt{S^2 - x^2}} \right)$$
(7)

Fig. 8 plots a hysteretic response of an example NSFDD calculated by Eq. (7). In this calculation, the value of *b* is taken as 16 kN, while the values for parameters in Eq. (7) are assumed to be identical to those values of mold coil spring used in Fig. 7. As indicated in Fig. 8(a), the horizontal force  $P_h$  generated by the gas springs provides stable negative stiffness in the O<sub>h2</sub>-A<sub>h2</sub> and O<sub>h2</sub>-B<sub>h2</sub> phases. At large displacements (e.g., A<sub>h2</sub> and B<sub>h2</sub>), negative stiffness remains substantial, because a high-level compressed force is maintained in the gas spring owing to the initial compressive force.

Fig. 8(b) depicts the friction force generated by  $P_v$ , i.e., the first term in Eq. (7). The hysteretic curve is similar as the Coulomb damping, excepting for a slight variation of the friction force at different displacement magnitudes. Such variation in the friction is induced by the changes of the pre-compressed force of the gas spring. At zero displacement (e.g., points  $O_{v3}$ ,  $O_{v4}$  in Fig. 8(b)), the pre-compressed force of the gas spring has the maximum values. At large displacement (e.g., points  $A_{v2}$ ,  $B_{v2}$ ,  $C_{v2}$ , and  $D_{v2}$  in Fig. 8(b)), the geometric vertical displacement of the lever arm results in a release of the pre-compressed deformation and corresponding force of the gas spring.
With a combination of the negative stiffness part and friction damping part, the total hysteretic
response of the NSFDD is shown in Fig. 8(c). Compared with Fig. 7(c), the pre-compressed gas
springs can ensure the NSFDD has robust mechanical properties at large displacements.



(generated by  $P_v$ )

NSFDD

Fig. 8 Force-displacement relationship of the NSFDD with pre-compressed gas springs

## 248 **3.2.3 Parametric analysis of NSFDD**

(generated by  $P_{\rm h}$ )

This section discusses the effect of various parameters on the two important design indices of the NSFDD, i.e., the negative stiffness and energy dissipation. The tangent stiffness k of the NSFDD is obtained by differentiation of Eq. (7) with respect to displacement x, given by:

$$k = -n \left( \frac{x^{2}}{\left(S^{2} - x^{2}\right)^{3/2}} + \frac{1}{\sqrt{S^{2} - x^{2}}} \right) \times \left( k_{g} \left( \Delta x - n \left(S - \sqrt{S^{2} - x^{2}}\right) \right) + b \right)$$

$$- \frac{n^{2} k_{g} x \left( \text{sgn}(\dot{x})u - \frac{x}{\sqrt{S^{2} - x^{2}}} \right)}{\sqrt{S^{2} - x^{2}}}$$
(8)

From Section 3.2.2 we know that the values of amplification coefficient *n* and the parameters (*b*,  $k_g$ , and  $\Delta x$ ) of gas springs affect the horizontal force component  $P_h$ , eventually influencing the negative stiffness value. From Eq.(7), it can be seen that the increase of the amplified vertical force  $P_v$  and friction coefficient results in a linear increase of energy dissipation. The length of rotation rod II influences its rotation angle in a given displacement, thus directly influencing the negative stiffness. The increase of the length *S* of rotation rob II results in a decrease of the rotation angle in a given displacement, and thus a decrease of negative stiffness. Therefore, the negative stiffnessis inversely proportional to the length *S*.

Fig. 9 depicts the influence of the length of rotating rod II (ranging from 60 to 100 mm) and the amplified vertical force  $P_v$  (ranging from 140 to 150 kN) on the stiffness (from Eq (8)) of the NSFDD at displacement magnitude 20 mm displacement. The figure further confirms the aforementioned influence of both factors. In practical design, the design value of negative stiffness can be achieved by adjusting those two factors. As indicated in Fig. 9, the length of the rotating rod II inversely affect the negative stiffness of the NSFDD, while the amplified vertical force is proportional to the negative stiffness values of the NSFDD.

Energy dissipation is another important index for the NSFDD. The energy dissipated in a 267 loading cycle can be calculated by integrating the NSFDD force along with the displacement. In 268 this calculation, the displacement magnitude is  $\pm 20$  mm, and the length of rotating rob II S = 80269 mm. The amplified vertical force (ranging from 140 to 150 kN) and friction coefficient (ranging 270 from 0.1 to 0.3) are taken as variables. As indicated in Fig. 10, an increase of both amplified 271 vertical force and friction coefficient leads to a linear increase of energy dissipation. The 272 273 adjustment of both factors can be considered, to achieve the designed target energy dissipation 274 capacity.

To conclude, the amplified vertical force  $P_v$  value linearly influences both the negative stiffness value and energy dissipation capacity of NSFDD, while the length of the rotating rod II and friction coefficient influence the negative stiffness value and energy dissipation capacity of the NSFDD, respectively. A strategic adjustment of three parameters should be performed to realize the design target values of the negative stiffness and energy dissipation of the NSFDD.

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Fig. 9 Influence on negative stiffness



Fig. 10 Influence on energy dissipation

## 280 **4 Experiments of the NSFDD**

# 281 4.1 Experimental specimen

The experimental specimen is composed of the pre-compressed spring system, top plate, base plate, restraint frame, friction plate, rotating rod (I and II), and two-consecutive lever (first-stage lever and second-stage lever), as depicted in Fig. 11. The pre-compressed spring system is fixed on the top plate. The pre-compressed gas spring force is applied to the first-stage lever through the track wheel, amplified by a two-consecutive lever, and eventually transmitted to the friction plate through the rotating rod. The geometric dimensions of the experimental specimen as shown in Fig. 11(c).



(a) Detailed 3D model (internal display)





(c) Geometric dimensions

# Fig. 11 Experimental specimen of the NSFDD

As shown in Fig. 12(a), when the lever is compressed by  $F_a$ , the rotating rod exerts an upward force  $F_b$  on the middle of the lever and the fulcrum exerts a downward force  $F_c$  on the lever. Therefore, the lever exerts the upward force  $F_d$  on the middle of the fulcrum and the restraint frame exerts the downward reaction force  $F_e$  on the ends of the fulcrum, as shown in Fig. 12(b). The concentrated forces at two fulcrums are eventually transferred to the restraint frame, and they are self-balanced. Therefore, the reaction force at the fulcrums does not affect the performance of the device and adjacent structural components.





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As shown in Fig. 13, the spring system is specifically designed to ensure the stable application 298 of a vertical force to the lever arm. Two ends of the gas spring are connected to the top plate and 299 a transmission plate by bolts. Four corners of the transmission plate are provided with linear 300 301 bearings which passed through smooth rods fixed on the top plate, ensuring that two plates (top 302 plate and transmission plate) are always parallel to each other during the movement. A track wheel is adopted to connect the transmission plate and the lever arm beneath it. The lever arm is placed 303 in the groove of the track wheel, and thus the level arm maintained stable contact by small rotation 304 305 of the track wheel when it moves up and down. The number of gas springs and the initial distance between the top plate and transmission plate can be adjusted to meet the design requirements of 306 the NSFDD mechanical properties. 307





## 309 4.2 Experimental results

Fig. 14 shows the photographs of the experimental specimen of the NSFDD. The restraint frame made of I-shaped steel beam is connected to the top and base steel plates by welding. The fulcrum of the lever arms are connected to the restraint frame. The friction plate consisted of a steel plate and four brake pads (its length, width, and thickness are all 100, 80, and 15 mm, respectively) fixed at its bottom surface. The brake pads are made of a phenolic resin composite mixed with copper flakes. The initial compressive force and stiffness of the gas spring are 16 kN and 120 kN/m, respectively. By applying the pre-compressed deformation of 100 mm, the gas

spring have a compressive force equal to 28 kN. The pre-compressed gas spring force is further 317 amplified 15 times by the two-consecutive lever. The kinetic friction coefficient between the brake 318 319 pad and the surface of the steel base plate was determined as 0.29 by friction coefficient measurement tests. The test was loaded at a loading velocity of 2 mm/s. In the friction coefficient 320 measurement tests, the size and shape of the brake pad were identical to those in the experimental 321 tests of the NSFDD. The base plate of the NSFDD specimen is fixed with the foundation beam of 322 the reaction frame through three steel boxes by welds and bolts. The specimen is loaded by a 323 horizontal actuator that is connected to the friction plate. The cyclic loading is displacement 324 controlled. Two loading cases are considered in this test. The displacement magnitude is  $\pm 20$  mm 325 for loading case 1 and  $\pm$  30 mm for loading case 2. For both loading cases, the loading is repeated 326 by 10 cycles. The horizontal force was measured by the load cell in the actuator, and the 327 displacement was measured by the string potentiometer. 328







(b) Display of the internal components of the NSFDD

329

Fig. 14 Photographs of experimental specimen of the NSFDD

Fig. 15 presents the hysteretic responses of the lateral force versus displacement relationship of the NSFDD specimen for two loading cases. The hysteretic curves clearly indicate a characteristic of combined negative stiffness and friction damping. During ten cycles of loading, the hysteretic response remained very stable without performance degradation. Due to the

amplification mechanism of the two-consecutive lever, the small-scaled NSFDD specimen could 334 reach the maximum lateral force of 150.9 and 174.5 kN for loading cases 1 and 2, respectively. 335 The NSFDD specimen achieved a high amount of energy dissipation per unit weight, which is 336 approximately 12.4 and 17.9 j/kg for loading cases 1 and 2. The efficiency of energy dissipation 337 338 is higher than the conventional negative stiffness device (e.g., the device in Reference [4] have the energy dissipation per unit weight of about 0.52 j/kg). Note that the negative stiffness 339 metamaterials can achieve even higher energy dissipation efficiency (e.g., the negative stiffness 340 honeycomb materials <sup>[52]</sup> have the energy dissipation per unit weight of 50 - 160 J/kg). 341

The hysteretic curves calculated using the theoretical model (i.e., Eq. (7)) are also plotted in 342 the Figure. The theoretical results correlated well with the experimental data. The energy 343 dissipation in one cycle calculated from the theoretical model was consistent with the experimental 344 results, with an average discrepancy of 2.2%. A slight difference of lateral forces was observed in 345 346 the large displacement of positive loading. In addition, at the transmission point from loading to unloading, slight displacement occurred in the experimental curves because small gaps existed 347 between the pins and the hole of the levers and rods. Such small gaps are not considered in the 348 theoretic model. 349



Fig. 15 Comparison of experimental and theoretical results of the NSFDD

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351 (Note: *E* denotes the energy dissipated in one cycle of loading, subscribes exp and theory denote

352 the results of the experimental test and theoretic model analysis, respectively.)

**553 5** Finite element analysis of the NSFDD

A finite element (FE) model is developed using ABAQUS software to simulate the behavior of the NSFDD. The objective of this FE analysis is to validate the assumptions used in the development of the mechanical model (e.g., ignoring the rotation angle of rotation rob I and the weight of the lever), and to simulate the behavior of large-capacity NSFDDs.

# 358 **5.1 Finite element model**

The FE model is used to represent the experimental specimen in Section 4. Except for the 359 pre-compressed gas spring, all components of the NSFDD is modeled by the solid element C3D8R, 360 as shown in Fig. 16. The steel material is simulated by a linear model because the steel does not 361 yield in the experimental tests. The elastic modulus and the Poisson's ratio of the steel are taken as 362  $2.06 \times 10^5$  MPa and 0.3, respectively. The nodes of the hole surface at C and K (see Fig. 16) are 363 constrained with the reference points that represented the fulcrum of the lever arms. Surface-to-364 surface discretization is achieved by the friction plate and the base plate ("hard" contact and the 365 366 tangential behavior are adopted). The four brake pads with actual geometric dimensions are 367 modeled in the friction plate bottom of the finite element model and the friction coefficient is set to 0.29. All DOFs of nodes of the base plate are fixed. 368

The pre-compressed gas spring is modeled using the "wire feature" in ABAQUS software. The parameters of the "wire feature" are assigned according to the mechanical properties of the gas spring, i.e., stiffness of 120 kN/m, initial compressive force of 16 kN, and the pre-compressed deformation of 100 mm. The nodes of the hole surface at M (see Fig. 16) are constrained with a reference point, and this reference point is connected to the simulated gas spring, such that the precompressed force is applied to the lever arm. Note that the simulation also considers the gravity loads of NSDFF self-weight. The horizontal cyclic displacement load is applied to the friction





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Fig. 16 Finite element model of the NSFDD with amplification mechanism

# 378 **5.2 FE analysis results**

The hysteretic curves of the NSFDD at loading displacement magnitude of  $\pm 20$  mm and  $\pm 30$ 379 mm is obtained by FE simulation, as presented in Fig. 17. The experimental results and theoretical 380 results using Eq. (7) are also plotted in the figures. It is indicated that the hysteretic curves of FE 381 382 simulation are nearly identical to the theoretical results, and both correlate well with the experimental data. Note that the theoretical model neglects the possible influence of the NSFDD 383 self-weight and the rotation angle of rotating rob I, while both of them are considered in the FE 384 385 simulation. The comparison of the results indicates that such neglection in the simplified theoretic model is reasonable, as it does not lead to errors in estimated hysteretic responses of the NSFDD. 386



Fig. 17 Comparison of FE analytical results with experimental and theoretical results

#### 388 5.3 Simulation of large-capacity NSFDD

The effect of influencing parameters is further investigated using the FE model. To realize large-capacity of devices that are commonly used in the building structures, three gas springs are included in the FE model, while the geometry and other parameters remained identical to those described in Section 5.1. Fig. 18(a) shows the hysteretic responses obtained by the FE simulation, where the maximum force reaches approximately 500 kN at 30 mm displacement.

394 Two key parameters are considered as the variables. One is the length of rotation rob II that primarily influences the negative stiffness value, and another is the friction coefficient that 395 primarily affects the friction force and energy dissipation. As shown in Fig. 18(b), when the length 396 of the rotating rod II is taken as 100, 200, and 300 mm, the tangent stiffness around zero 397 displacement for the NSFDD model is -13.32, -6.66, and -4.44 kN/mm, respectively. The length 398 of the rotating rod inversely affects the negative stiffness of the NSFDD. Similarly, three values 399 for the friction coefficient  $\mu$  is considered, i.e.,  $\mu = 0.145, 0.29, 0.435$ . As shown in 18(c), the 400 NSFDD force increase along with an increase of the  $\mu$  value. Further calculation of the area 401 402 enveloped by the hysteretic curve indicates that the increase of energy dissipated in a loading cycle 403 is linearly proportional to the increase of the  $\mu$  value. The FE simulation further validates the findings in 3.2.3. 404



(a) Results of the large-capacity NSFDD





(c) Effect of friction coefficient

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Fig. 18 FE analysis of hysteresis curves of large-capacity NSFDD

## 406 Conclusions

This paper proposes a novel negative stiffness friction damping device (NSFDD). This NSFDD can achieve large force capacity owing to use of the amplifying mechanism and precompressed gas springs. The following major conclusions are drawn from this study.

(1) A two-consecutive level and pre-compressed gas springs are used in the NSFDD. A twoconsecutive level can significantly amplify the damping force. The gas spring is beneficial for
maintaining stable negative stiffness and friction force in a large displacement of the NSFDD.

(2) A mechanical model of the NSFDD is proposed. The parametric analysis using the mechanical model indicates that the amplified vertical force is proportional to the negative stiffness and energy dissipation capacity of the NSFDD. The length of the rotating rod inversely affects the negative stiffness of the NSFDD, while the friction coefficient linearly influences energy dissipation capacity.

(3) The experimental tests and finite element (FE) model of the NSFDD subjected to lateral cyclic loads are conducted to examine its hysteretic behavior. The NSFDD specimen exhibits an obvious negative stiffness and stable energy dissipation, verifying the effectiveness of the proposed NSFDD. The hysteretic curves obtain from the FE model correlate well with the experimental tests data.

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